Iterative Review (Plagiarism), Mirror–Dual Universal Set Framework and Observer Selection Functions: Axioms, Categorical Semantics, Gaussian Fixed Points, and Computational Boundaries

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Abstract

We present an analytic formalization of a "mirror–dual" foundations program centered on two interacting set–like universes and an *Observer Selection Function* (OSF). The motivating summary text asserts a transformation-closure slogan $U(U(U)) = U$, proposes that OSFs become Gaussian by Central Limit Theorem (CLT) and iterative mirroring, sketches an embedding into Grothendieck universes and topos theory, and further suggests computational consequences such as halting decidability and a collapse of complexity classes. We separate definitional content from provable content. First, we define a mirroring operator on probability laws and prove that, under finite second moment, the only exact fixed points are Gaussian. Second, we give a precise Grothendieck/topos semantics in which "internal" and "external" worlds are connected by a geometric morphism, and we define OSFs as stochastic natural transformations (Markov kernels) compatible with that morphism. Third, we prove the classical diagonal obstruction: no total *computable* halting decider exists, so any universal halting claim must be interpreted via restriction, oracle power, or a non-classical internal notion of computability. We conclude with a clean list of axioms, theorems, and open problems required to turn the mirror–dual program into a rigorous framework.

Contents

1 Introduction

The motivating summary proposes two interacting "universal" structures: a self-containing world US and a non-self-containing world UN, connected by mirror dynamics, from which observer selection mechanisms (OSFs) are said to emerge as Gaussian functions via CLT and a recursive mirroring process. It further sketches a set-theoretic embedding via Grothendieck universes and a categorical embedding via topos theory and natural transformations, and it suggests far-reaching computational consequences (halting decidability and P vs NP resolution).

This paper has one goal: replace slogans with typed objects, explicit axioms, and analytic theorems. We proceed in three layers:

L1. Axioms and objects. We define US, UN as universes (set-theoretic or categorical) and specify a *closure operator* U whose slogan $U(U(U)) = U$ becomes an algebraic law.

L2. OSFs and mirroring. We define OSFs as probability kernels and formalize "mirroring" as an operator on laws. Under a finite-variance assumption, we prove that Gaussian laws are exactly the fixed points of the mirroring operator.

L3. Computation. We state and prove the standard impossibility theorem for computable halting deciders. Consequently, any universal "halting is decidable" claim must change the computational model (restriction/oracle/non-classical semantics). We make these options explicit.

Throughout, we use only analytic arguments (no numerical methods). Limits appear only as exact statements of convergence in distribution or topology, as in standard probability theory.

2 Preliminaries

2.1 Probability laws and characteristic functions

Let $\mathcal{P}(\mathbb{R})$ denote Borel probability measures on $\mathbb{R}$. Let $\mathcal{P}_2(\mathbb{R}) \subset \mathcal{P}(\mathbb{R})$ denote those with finite second moment. For $\mu \in \mathcal{P}(\mathbb{R})$, its characteristic function is

$$\varphi_\mu(t) = \int_{\mathbb{R}} e^{itx}\, d\mu(x).$$

If $X \sim \mu$, we write $\mathcal{L}(X) = \mu$.

2.2 Grothendieck universes and topoi

We assume familiarity with the following standard notions.

Definition 2.1 (Grothendieck universe)**.** A set U is a Grothendieck universe if it is transitive and closed under pairing, unions of indexed families, power sets, and function sets. Concretely: if $x, y \in U$ then $\{x, y\} \in U$; if $x \in U$ then $\mathcal{P}(x) \in U$; if $I \in U$ and $x_i \in U$ for all $i \in I$ then $\bigcup_{i\in I} x_i \in U$; if $x, y \in U$ then $y^x \in U$.

Definition 2.2 (Grothendieck topos, geometric morphism)**.** A Grothendieck topos is a category equivalent to sheaves $\mathbf{Sh}(\mathcal{C}, J)$ on a site. A geometric morphism $f : \mathcal{E} \to \mathcal{F}$ is an adjoint pair $(f^{,f})$ where $f^{:\mathcal{F}\to\mathcal{E}}$ is left exact (preserves finite limits) and $f^{\dashv f}$.

3 Axiomatizing Mirror–Dual Universes and Closure

The summary treats "transformation closure" as the central principle and writes the slogan

$$U(U(U)) = U.$$

To make this meaningful, one must specify the type of U and its law.

3.1 Closure operator: minimal algebraic content

Definition 3.1 (Closure operator)**.** Let X be a set (or object in a category) and let $U : \mathcal{P}(X) \to \mathcal{P}(X)$ be an operator. We say U is a closure operator if for all $A, B \subseteq X$,

$$A \subseteq U(A), \qquad A \subseteq B \implies U(A) \subseteq U(B), \qquad U(U(A)) = U(A).$$

Remark 3.1. The slogan $U(U(U)) = U$ is automatically satisfied if U is idempotent on its relevant domain. Indeed $U(U(U(A))) = U(U(A)) = U(A)$. Thus, as a stand-alone statement, the slogan is algebraically weak: its content lies entirely in what U acts on and what structures U is meant to close.

3.2 Two-universe schema

We present two compatible formalisms: a set-theoretic schema and a categorical schema. They can be combined, but are conceptually distinct.

3.2.1 Set-theoretic schema

Axiom 3.1 (Large-universe assumption)**.** There exist Grothendieck universes $\mathsf{US} \subset \mathsf{UN}$.

Remark 3.2. A standard sufficient condition is the existence of an inaccessible cardinal κ with $\mathsf{US} = V_\kappa$, and a larger inaccessible λ with $\mathsf{UN} = V_\lambda$. This is a *relative consistency* viewpoint: the mirror–dual program is then at least as consistent as ZFC plus these large-cardinal assumptions.

3.2.2 Topos-theoretic schema

Axiom 3.2 (Internal/external worlds)**.** There exist Grothendieck topoi $\mathcal{E}$ (internal) and $\mathcal{F}$ (external) and a geometric morphism $f : \mathcal{F} \to \mathcal{E}$.

We interpret $\mathcal{E}$ as a semantic universe for "self-contained" reasoning, and $\mathcal{F}$ as an externalized semantic universe where truth values and observations can depend on context (as in sheaf semantics). The inverse image functor $f^{:\mathcal{E}\to\mathcal{F}}$ transports internal objects to external ones.

4 Observer Selection Functions as Stochastic Natural Transformations

The summary suggests OSFs should be treated as functors or natural transformations between internal and external structures. To incorporate probability (and thereby speak about Gaussians), the transformation must land in a space of laws.

4.1 Markov kernels

Definition 4.1 (Markov kernel)**.** Let (X, Σ_X), (Y, Σ_Y) be measurable spaces. A Markov kernel $K : X \times \Sigma_Y \to [0, 1]$ satisfies: (i) for each x, $A \mapsto K(x, A)$ is a probability measure on Y; (ii) for each A, $x \mapsto K(x, A)$ is measurable.

Kernels encode "selection" probabilistically: given a state x, the kernel returns a law over observations.

4.2 OSF as a kernel on a measurable object

Definition 4.2 (OSF on a measurable object)**.** Fix a measurable space X. An *Observer Selection Function* on X is a Markov kernel

$$\mathsf{OS} : X \rightsquigarrow X$$

(or more generally $\mathsf{OS} : X \rightsquigarrow Y$ to an observation space Y).

4.3 OSF as a stochastic natural transformation

To align with the categorical intent, consider functors $F, G : \mathcal{E} \to \mathcal{F}$. A purely deterministic natural transformation $\eta : F \Rightarrow G$ has components $\eta_X : F(X) \to G(X)$. A probabilistic generalization replaces arrows with kernels.

Definition 4.3 (Stochastic natural transformation (informal))**.** Assume a probability monad Prob on $\mathcal{F}$ (e.g. a Giry-style construction on suitable measurable objects). A *stochastic natural transformation* is a family of arrows

$$\mathsf{OS}_X : F(X) \longrightarrow \mathsf{Prob}(G(X))$$

natural in $X \in \mathcal{E}$, i.e. satisfying the usual commutative squares after lifting morphisms through Prob.

Remark 4.1. This definition is deliberately schematic: constructing Prob internally to a general topos requires choices (valuations, probability objects, or a specific class of measurable objects). The point is type correctness: "OSF assigns probabilities" must be represented as a morphism into a probability object.

5 Mirroring Dynamics and the Gaussian Fixed-Point Theorem

We now formalize the mirroring process claimed to "refine OSFs" and "converge to a Gaussian". We do this in a standard analytic setting: laws on $\mathbb{R}$.

5.1 The mirroring operator

Definition 5.1 (Mirroring operator on laws)**.** Define $M : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ as follows: for $\mu \in \mathcal{P}_2(\mathbb{R})$, let X, Y be i.i.d. with $\mathcal{L}(X) = \mathcal{L}(Y) = \mu$ and set

$$M(\mu) := \mathcal{L}\left(\frac{X+Y}{\sqrt{2}}\right).$$

Remark 5.1. Iterating M yields

$$M^n(\mu) = \mathcal{L}\left(\frac{X_1 + \cdots + X_{2^n}}{\sqrt{2^n}}\right)$$

with $(X_i)_{i\geq 1}$ i.i.d. $\sim \mu$. Thus "repeated mirroring" is exactly an averaging-and-rescaling scheme.

5.2 Gaussian fixed points are the only finite-variance fixed points

Theorem 5.1 (Gaussian fixed-point rigidity)**.** *Let $\mu \in \mathcal{P}_2(\mathbb{R})$ have mean 0. If $M(\mu) = \mu$, then μ is a centered Gaussian measure $N(0, \sigma^2)$ for some $\sigma^2 \geq 0$.*

Proof. Let $X \sim \mu$ and $Y \sim \mu$ be independent. The condition $M(\mu) = \mu$ is equivalent to

$$X \stackrel{d}{=} \frac{X+Y}{\sqrt{2}}.$$

Let $\varphi(t) = \mathbb{E}[e^{itX}]$ be the characteristic function of μ. By independence, the characteristic function of $(X+Y)/\sqrt{2}$ equals $\varphi(t/\sqrt{2})^2$, hence the fixed-point condition gives

$$\varphi(t) = \varphi(t/\sqrt{2})^2 \qquad \text{for all } t \in \mathbb{R}.$$

Since φ is continuous and $\varphi(0) = 1$, there exists a neighborhood of 0 on which $\varphi \neq 0$. The functional equation implies $\varphi(t) \neq 0$ for all t because $\varphi(t) = \varphi(t/2^{n/2})^{2^n}$ and $\varphi(t/2^{n/2}) \to \varphi(0) = 1$. Thus $f(t) = \log \varphi(t)$ is well-defined and continuous on $\mathbb{R}$. Taking logs yields

$$f(t) = 2f(t/\sqrt{2}).$$

Finite second moment implies $\varphi \in C^2(\mathbb{R})$, hence $f \in C^2(\mathbb{R})$. Differentiate twice:

$$f'(t) = \sqrt{2}\, f'(t/\sqrt{2}), \qquad f''(t) = f''(t/\sqrt{2}).$$

Iterating the second identity gives $f''(t) = f''(t/2^{n/2}) \to f''(0)$ as $n \to \infty$ by continuity, hence $f''(t) \equiv f''(0)$ is constant. With mean 0, $\varphi'(0) = i\mathbb{E}[X] = 0$, so $f'(0) = 0$ and $f(0) = 0$. Therefore $f(t) = \frac{f''(0)}{2} t^2$. Finally $f''(0) = \varphi''(0) - (\varphi'(0))^2 = -\mathbb{E}[X^2] = -\sigma^2$, giving

$$\varphi(t) = e^{-\sigma^2 t^2/2},$$

the characteristic function of $N(0, \sigma^2)$. □

Corollary 5.1 (Exact OSF Gaussianity as a fixed-point principle)**.** *If an OSF on $\mathbb{R}$ is modeled by a law $\mu \in \mathcal{P}_2(\mathbb{R})$ and the mirror refinement axiom is $\mu = M(\mu)$, then μ is Gaussian. No additional approximation or asymptotic reasoning is required.*

5.3 CLT as attraction of iterated mirroring

Theorem 5.2 (CLT via mirroring iterates (classical))**.** *Let $\mu \in \mathcal{P}_2(\mathbb{R})$ have mean 0 and variance $\sigma^2 > 0$. Then*

$$M^n(\mu) \Rightarrow N(0, \sigma^2) \quad \text{as } n \to \infty,$$

where $\Rightarrow$ denotes convergence in distribution.

Remark 5.2. This statement is the standard CLT expressed in the dyadic averaging scheme naturally induced by M. The fixed-point theorem above identifies the *only* finite-variance law invariant under one mirroring step.

5.4 What "Gaussian invariance under transformations" can mean

A Gaussian law is stable under affine maps: if $X \sim N(\mu, \sigma^2)$ then $aX + b \sim N(a\mu + b, a^2\sigma^2)$. If the framework demands an OSF invariant *as a family* under coordinate changes, the correct formulation is: OSFs transform covariantly (pushforward) and the family of Gaussian laws is closed under that action.

6 Topos Semantics for Mirror Duality

We now supply a coherent categorical reading of "self-contained" vs "externally dependent" worlds.

6.1 Internal and external topoi

Let $\mathcal{E}$ be an "internal" topos, and $\mathcal{F} = \mathbf{Sh}(\mathcal{C}, J)$ an "external" sheaf topos. A geometric morphism $f : \mathcal{F} \to \mathcal{E}$ provides $f^{:\mathcal{E} \to \mathcal{F}}$, transporting internal objects to their external realizations.

Definition 6.1 (Mirror interface)**.** A *mirror interface* consists of a geometric morphism $f : \mathcal{F} \to \mathcal{E}$ together with a specified class $\mathscr{X} \subset \mathrm{Ob}(\mathcal{E})$ of "observable" objects and, for each $X \in \mathscr{X}$, a probability object $\mathsf{Prob}(f^{X)\in\mathcal{F}}$.

6.2 OSFs as selection kernels across the interface

Definition 6.2 (OSF across internal/external worlds)**.** Given a mirror interface, an OSF is a family of morphisms

$$\mathsf{OS}_X : f^{X \longrightarrow \mathsf{Prob}(f^X),} \qquad {}^{X \in \mathscr{X},}$$

natural in X (in the stochastic sense).

Remark 6.1. This definition realizes the slogan "OS selects from internal structure into external observation" as a typed construction. Any additional physics interpretation must be expressed by choosing $\mathscr{X}$, the site $(\mathcal{C}, J)$, and the probability object $\mathsf{Prob}(-)$.

6.3 Placing the mirroring operator inside the topos

If $X \in \mathscr{X}$ carries an additive structure (e.g. an internal real line object), then $\mathsf{Prob}(f^X)$ can support a convolution-like operation, allowing one to define an internal version of the mirroring operator M. The Gaussian fixed-point theorem then becomes an assertion about fixed points of that operator, subject to finite-variance hypotheses expressed internally (e.g. second moment as a global section).

7 Computability and Complexity: What Follows, What Cannot

The motivating summary asserts that transformation closure eliminates undecidability and yields polynomial-time solutions of NP problems. Any such statement must specify the computational model.

7.1 Diagonal obstruction: no total computable halting decider

Theorem 7.1 (Turing's halting impossibility)**.** *There is no total computable function $H(p, x) \in \{0, 1\}$ such that $H(p, x) = 1$ iff program p halts on input x.*

Proof. Assume such an H exists. Define a program D which on input p does: if $H(p, p) = 1$ then loop forever, else halt immediately. Consider $D(D)$. If $H(D, D) = 1$, then $D(D)$ loops, contradicting that H predicts halting. If $H(D, D) = 0$, then $D(D)$ halts, contradicting H's prediction. Hence no such total computable H exists. □

Corollary 7.1 (Interpretation constraint)**.** *If a framework asserts a universal halting decider, then at least one of the following must hold:*

(a) the "decider" is not *computable in the classical sense (oracle/hypercomputation);*

(b) the claim is restricted to a decidable subclass of programs (bounded-time, total languages, finite-state models);

(c) the internal notion of computability is non-classical and must be specified by semantics (e.g. realizability/topos logic).

7.2 What it would take to deduce $P = NP$

Any proof that $P = NP$ must produce a uniform polynomial-time algorithm for an NP-complete problem (e.g. SAT). Transformation language alone is insufficient; one must control encoding size and runtime.

Problem 7.1 (Algorithmic sufficiency for a "closure solves SAT" claim)**.** To prove SAT $\in P$, it suffices to provide:

(i) a polynomial-time computable map $x \mapsto \Phi(x)$ from SAT instances to instances of a target problem T;

(ii) a polynomial bound $|\Phi(x)| \leq \mathrm{poly}(|x|)$;

(iii) a polynomial-time algorithm deciding $T(\Phi(x))$;

(iv) correctness: $x \in \mathrm{SAT} \iff \Phi(x) \in T$.

Any "transformation closure" mechanism must realize (i)–(iv) *explicitly.*

Remark 7.1. If a framework permits non-uniform "eigenstate" advice depending on input length, it risks proving at best NP $\subseteq$ P/poly, not $P = NP$. The uniformity condition must be stated.

7.3 A consistent path: restricted halting and polynomial closure

One consistent (and provable) program is:

R1. Choose a restricted language class $\mathscr{L}$ (e.g. total, structurally recursive programs).

R2. Prove an internal closure theorem: every computation in $\mathscr{L}$ admits a normal form (a "fixed point") computed in polynomial time.

R3. Interpret OSF "eigenstates" as normal forms, not as omniscient solutions to arbitrary computations.

This retains a meaningful "closure yields decidability" thesis without contradicting Turing's theorem.

8 Fractional Infinities and Eigenchoice: Formal Reconstruction

The summary text proposes (i) a "continuous infinity spectrum" ("fractional infinities") and (ii) an "eigenchoice" principle intended to replace unconstrained choice. In classical foundations, these phrases cannot be interpreted literally as statements about *cardinalities* without changing the underlying notion of size and the notion of admissible selection. This section provides two rigorous replacements:

F1. A *graded size* (or *size spectrum*) replacing cardinality by an analytic invariant taking values in a continuum.

F2. A *structured selection* principle (eigenchoice) replacing arbitrary choice by selection compatible with specified structure (measurability, continuity, naturality, or internal logic).

8.1 Cardinality is discrete; graded size can be continuous

Cardinality is a complete invariant of bijection, and it is rigidly discrete in the sense that it lives in the well-ordered class of cardinals and satisfies Cantor's theorem $|X| < |\mathcal{P}(X)|$ for every set X. Thus, any claim of a "continuous" family interpolating between $\aleph_0$ and $2^{\aleph_0}$ is *not* a statement in the language of classical cardinal arithmetic unless one changes what "size" means.

Definition 8.1 (Ordered size codomain)**.** An *ordered size codomain* is a totally ordered commutative semiring $(\Lambda, +, \cdot, \leq)$ equipped with a topology in which Λ contains a connected subset (e.g. an interval) so that "continuous spectra" are meaningful.

Definition 8.2 (Graded size functional)**.** Let $\mathscr{S}$ be a class of mathematical objects (sets, measurable spaces, topological spaces, groups, etc.). A *graded size* on $\mathscr{S}$ is a function

$$s : \mathscr{S} \to \Lambda$$

to an ordered size codomain Λ, intended to satisfy axioms appropriate to the structure.

8.2 Axioms for a size spectrum

The specific axioms depend on the category of objects. The following are common and purely analytic.

Axiom 8.1 (Monotonicity under embeddings)**.** If $A, B \in \mathscr{S}$ and there exists an injective structure-preserving map $A \hookrightarrow B$, then $s(A) \leq s(B)$.

Axiom 8.2 (Additivity on disjoint unions)**.** If $A, B \in \mathscr{S}$ and $A \sqcup B$ exists in $\mathscr{S}$ as a disjoint union, then $s(A \sqcup B) = s(A) + s(B)$.

Axiom 8.3 (Multiplicativity on products)**.** If $A, B \in \mathscr{S}$ and $A \times B$ exists in $\mathscr{S}$ as a product, then $s(A \times B) = s(A) \cdot s(B)$.

Axiom 8.4 (Continuity under directed limits (when applicable))**.** If $(A_i)_{i \in I}$ is a directed system in $\mathscr{S}$ with colimit A, then $s(A) = \sup_{i \in I} s(A_i)$ whenever the supremum exists in Λ.

Remark 8.1. These axioms are intentionally schematic: they describe how "size" behaves under the constructors that are actually present in $\mathscr{S}$. They define a *continuum-valued* size theory without conflicting with Cantor's theorem, because s is not required to coincide with cardinality.

8.3 Examples that produce a continuous spectrum

The mirror–dual program becomes concrete once one chooses $\mathscr{S}$ and s. Examples include:

(a) **Measure size:** $\mathscr{S}$ measurable sets in a fixed measure space; $s(A) = \mu(A) \in [0, \infty]$.

(b) **Dimension size:** $\mathscr{S}$ metric spaces; $s(X) = \dim_H(X) \in [0, \infty]$ (Hausdorff dimension).

(c) **Entropy size:** $\mathscr{S}$ probability distributions; $s(\mu) = H(\mu) \in [0, \infty]$ (Shannon entropy) or differential entropy where defined.

(d) **Categorical rank:** $\mathscr{S}$ objects in a topos or accessible category; $s(X)$ a rank/height in an ordered class that can embed intervals when restricted to a subcollection.

Remark 8.2. When the summary says the Continuum Hypothesis becomes "irrelevant," the only rigorous reading is: CH is a question about *cardinals*; a graded size theory is about *different invariants* and therefore does not address CH directly. Instead, one obtains a continuum of sizes by construction.

8.4 No-go: continuous graded size cannot equal cardinality everywhere

Proposition 8.1 (Cardinality cannot be continuously interpolated)**.** *Let $\mathscr{S}$ contain all sets, and suppose $s : \mathscr{S} \to \Lambda$ satisfies: (i) $s(A) = s(B)$ whenever $A \cong B$ (bijection invariance), (ii) $s(A) < s(\mathcal{P}(A))$ (Cantor monotonicity reflected in Λ), and (iii) Λ contains a connected subset with the order topology. Then s cannot coincide with classical cardinality on all sets while also admitting a "continuous spectrum" that interpolates between $\aleph_0$ and $2^{\aleph_0}$ in the sense of order-density.*

Proof. If s coincides with cardinality on all sets, its values on infinite sets are well-ordered by $\leq$, hence contain no nontrivial connected subsets in the order topology. Any order-dense interpolation between two distinct cardinal values contradicts well-order discreteness. Therefore a continuous spectrum forces s to be a different invariant than cardinality on at least some objects (typically, all sufficiently large ones). □

9 Eigenchoice as Structured Selection

The summary proposes an "eigenchoice" principle: selection governed by admissible structure rather than arbitrary choice. Formally, choice principles are statements about *splittings of surjections* or *sections of set-valued maps.* Eigenchoice is then "choice with constraints."

9.1 Choice as splitting surjections

Definition 9.1 (Choice function on a family)**.** Let $\mathcal{A}$ be a family of nonempty sets. A *choice function* on $\mathcal{A}$ is a function $\chi : \mathcal{A} \to \bigcup_{A \in \mathcal{A}} A$ such that $\chi(A) \in A$ for all $A \in \mathcal{A}$.

Definition 9.2 (Section formulation)**.** Given a surjection $\pi : E \to B$, a *section* is a function $s : B \to E$ such that $\pi \circ s = \mathrm{id}_B$. A family-choice function corresponds to a section of the canonical projection from a disjoint union of fibers.

9.2 Eigenchoice: selection compatible with structure

Definition 9.3 (Eigenchoice schema)**.** Fix a category $\mathcal{K}$ of structured objects (measurable spaces, topological spaces, etc.). An *eigenchoice* principle asserts the existence of sections (or selectors) that are morphisms in $\mathcal{K}$ whenever the underlying surjection (or set-valued map) is a morphism in $\mathcal{K}$.

Remark 9.1*.* Eigenchoice is not "Choice without paradox" by proclamation; it is a *restricted* choice principle: one demands the selector be measurable/continuous/natural/internal, which is only possible under hypotheses. This aligns with the intent to avoid pathologies such as non-measurable selections, but it also necessarily weakens full Choice.

9.3 Canonical analytic instances of eigenchoice

Theorem 9.1 (Measurable selection (prototype eigenchoice theorem))**.** *Let X be a standard Borel space and let $F : X \to 2^Y$ be a set-valued map into nonempty closed subsets of a Polish space Y, with Borel graph. Then there exists a Borel measurable selector $f : X \to Y$ with $f(x) \in F(x)$.*

Remark 9.2*.* This theorem (and its variants) formalizes the slogan "selection from S" in a way that is compatible with structure. It illustrates the correct pattern: eigenchoice is a *theorem under hypotheses* or an *axiom for a restricted class*, not a universal replacement for Choice in full generality.

9.4 Eigenchoice in topos semantics

In a topos $\mathcal{F}$, internal logic provides a setting where "choice" becomes the existence of sections for internal epimorphisms. One can therefore define eigenchoice as:

Definition 9.4 (Internal eigenchoice (topos form))**.** Let $\mathcal{F}$ be a topos with a distinguished class of epimorphisms $\mathscr{E}$. *Internal eigenchoice for $\mathscr{E}$* is the axiom that every $e : E \twoheadrightarrow B$ in $\mathscr{E}$ admits a section $s : B \to E$ satisfying additional definability constraints (e.g. naturality across a specified subtheory).

10 Synthesis: A Minimal Rigorous Core for MDUSF/OSF

We now state a minimal core of axioms and theorems that (i) matches the categorical intent, (ii) yields a nontrivial analytic consequence (Gaussian rigidity), and (iii) respects classical computational impossibility theorems unless one explicitly changes semantics.

10.1 Core data: two worlds and an observation interface

Axiom 10.1 (Mirror interface)**.** There exist Grothendieck topoi $\mathcal{E}, \mathcal{F}$ and a geometric morphism $f : \mathcal{F} \to \mathcal{E}$. Fix a class $\mathscr{X} \subset \mathrm{Ob}(\mathcal{E})$ of observable objects.

Axiom 10.2 (Probability object)**.** There exists a probability-object constructor Prob on the externalizations f^X for $X \in \mathscr{X}$, sufficient to interpret Markov kernels as morphisms into $\mathsf{Prob}(f^X)$.

10.2 OSF as a stochastic natural transformation

Axiom 10.3 (OSF family)**.** An Observer Selection Function is a family of morphisms

$$\mathsf{OS}_X : f^{X \longrightarrow \mathsf{Prob}(f^X),} \qquad {}^{X \in \mathscr{X},}$$

natural in X (in the stochastic sense).

10.3 Mirror refinement as an operator on laws

For a distinguished observable $X_0 \in \mathscr{X}$ whose externalization admits an internal additive structure corresponding externally to $(\mathbb{R}, +)$, one obtains an induced action of OSFs on probability laws on $\mathbb{R}$. We state the refinement as an explicit law-level operator.

Axiom 10.4 (Mirroring operator on $\mathcal{P}_2(\mathbb{R})$)**.** There exists an operator $M : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ such that for $\mu \in \mathcal{P}_2(\mathbb{R})$,

$$M(\mu) = \mathcal{L}\left(\frac{X+Y}{\sqrt{2}}\right) \quad \text{for i.i.d. } X, Y \sim \mu.$$

10.4 Gaussian rigidity and stability

Theorem 10.1 (Gaussian rigidity for mirror-stable observers)**.** *Let $\mu \in \mathcal{P}_2(\mathbb{R})$ have mean 0. If $M(\mu) = \mu$, then $\mu = N(0, \sigma^2)$ for some $\sigma^2 \geq 0$.*

Theorem 10.2 (CLT as attraction under mirror iterates)**.** *Let $\mu \in \mathcal{P}_2(\mathbb{R})$ have mean 0 and variance $\sigma^2 > 0$. Then $M^n(\mu) \Rightarrow N(0, \sigma^2)$.*

Remark 10.1. These are exact analytic statements. "Gaussian OSF" is therefore a theorem *only* under the hypothesis that the OSF law is mirror-stable (fixed point) or asymptotically mirror-attracted (CLT regime) in the finite-variance category. Without finite variance, other stable laws may appear, so finite second moment is a genuine structural assumption, not decoration.

11 Computability and Complexity: Explicit Boundaries and Consistent Extensions

The summary suggests that transformation closure yields (i) a universal halting decider and (ii) polynomial-time solutions for NP problems. In classical computation, these claims require explicit scope restrictions or a strictly stronger model of computation.

11.1 Diagonal obstruction

Theorem 11.1 (No total computable halting decider)**.** *There is no total computable function $H(p, x) \in \{0, 1\}$ that decides whether program p halts on input x.*

Proof. Assume such H exists. Define a program D that on input p halts iff $H(p, p) = 0$, and loops otherwise. Evaluating $D(D)$ yields a contradiction in both cases $H(D, D) = 1$ and $H(D, D) = 0$. $\square$

Corollary 11.1 (Interpretation trichotomy)**.** *Any universal halting-decider claim must be interpreted as at least one of:*

(a) ***Restriction:*** *decidability for a proper subclass of programs;*

(b) ***Oracle/hypercomputation:*** *a non-Turing computational power is assumed explicitly;*

(c) ***Non-classical semantics:*** *"computable" is an internal notion in a specified semantic universe (e.g. a realizability topos), and must be defined and related back to external computation with care.*

11.2 What must be exhibited to claim $P = NP$

Problem 11.1 (Uniform polynomial-time content requirement)**.** To claim $P = NP$, it suffices (and is necessary in practice) to present a *uniform* polynomial-time algorithm for an NP-complete language such as SAT. Any "closure" mechanism must therefore provide:

(i) a polynomial-time computable encoding $x \mapsto \Phi(x)$;

(ii) a polynomial bound on $|\Phi(x)|$ in $|x|$;

(iii) a polynomial-time decider for the target property of $\Phi(x)$;

(iv) correctness equivalence $x \in \text{SAT} \iff \Phi(x) \in T$.

Absent (i)–(iv), the statement is not a complexity result but a re-description.

11.3 Consistent computational reading of "closure"

A consistent and testable reading is:

Axiom 11.1 (Restricted closure computability)**.** Fix a decidable computation class $\mathscr{L}$ (e.g. total, structurally recursive programs or bounded-time machines). Assume a closure operator U produces normal forms for computations in $\mathscr{L}$ and that the normal form is computable within the resource bounds defining $\mathscr{L}$.

Remark 11.1. This retains a meaningful "closure gives decidability" thesis while respecting the diagonal obstruction: one proves normal-form theorems for a controlled computation class rather than claiming omniscience for all programs.

12 Open Problems and Required Constructions

The mirror–dual program becomes rigorous only after completing the following constructions.

Problem 12.1 (Concrete probability object in the chosen $\mathcal{F}$)**.** Choose the class of external objects supporting probability (measurable objects, locales, etc.) and construct Prob with enough functoriality to interpret kernels and stochastic naturality.

Problem 12.2 (Internal second moment and finite variance)**.** Express "finite second moment" internally to $\mathcal{F}$ and prove it is preserved under the mirroring operator.

Problem 12.3 (A nontrivial closure operator U)**.** Specify U as an idempotent endofunctor/monad/comonad or as a closure operator on a lattice of predicates such that: (i) U is not simply the identity, (ii) U interacts coherently with f and Prob, and (iii) "mirror stability" can be expressed as a U-fixed point condition.

Problem 12.4 (Eigenchoice class and selector theorems)**.** Select a concrete eigenchoice class $\mathscr{E}$ (measurable/continuous/internal epimorphisms) and prove selector existence (or state it as an axiom) with explicit consistency strength and limitations.

Problem 12.5 (Fractional infinity as graded size)**.** Choose $\mathscr{S}$ and construct a graded size $s : \mathscr{S} \to \Lambda$ satisfying monotonicity/additivity/product laws and mirror invariance where appropriate. Prove nontrivial results (e.g. classification of s-fixed points under mirror dynamics).

Problem 12.6 (Computational semantics)**.** Fix one of the trichotomy options (restriction/oracle/non-classical semantics) and supply:

(a) definitions of programs, executions, and halting in that semantics;

(b) a translation/forgetful functor back to classical computation (if claimed);

(c) precise statements replacing "halting is solved" and "$P = NP$" with correct scope.

13 Conclusion

We distilled a minimal analytic core consistent with standard mathematics: a two-world interface via Grothendieck/topos semantics, OSFs as structured probability-valued transformations, and mirroring as an averaging-and-rescaling operator on laws. Within this core, Gaussianity is not a slogan: under finite second moment, it is a rigidity theorem for mirror-fixed laws and an attraction theorem under iterated mirroring (the CLT scheme). Claims that exceed this (universal halting decidability, $P = NP$) cannot be obtained in classical computation without explicit restrictions or a strictly stronger semantics. The program therefore reduces to concrete construction tasks: specify the probability object, specify eigenchoice, specify graded size, and specify computational semantics. Once those are fixed, the mirror–dual framework becomes a legitimate theorem-producing theory rather than a bundle of metaphors.

References

[1] P. Billingsley, *Probability and Measure*, Wiley.

[2] W. Feller, *An Introduction to Probability Theory and Its Applications*, Wiley.

[3] S. Mac Lane, *Categories for the Working Mathematician*, Springer.

[4] P. T. Johnstone, *Sketches of an Elephant: A Topos Theory Compendium*, Oxford University Press.

[5] M. Sipser, *Introduction to the Theory of Computation*, Cengage.

[6] A. S. Kechris, *Classical Descriptive Set Theory*, Springer.

[7] *Promising Theory of Everything Summary* (uploaded PDF).